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A study on solving Assignment Problem

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Abstract: *The topic of assignment is a critical problem in mathematics and is further explored in the real physical world. We try to implement a replacement method during this paper to solve assignment problems with algorithm and solution steps. By using new method and computing by existing two methods, we analyse a numerical example, also we compare the optimal solutions between this new method and two current methods. A standardized technique, simple to use to solve assignment problems, may be the proposed method.*

Keywords: *Assignment issue, HA-method, assignment method of Matrix one, Proposed method, Optimization.*

I. Introduction

We often encounter situations during which we have we've got to assign n jobs to n workers. All n workers are capable of doing all jobs, but with a varying cost. Hence our task is to search out the simplest possible assignment that offers maximum efficiency and minimum cost. Example, assigning activities to students, subjects to teachers, different routes of pizza delivery boys, salesmen to different regions, jobs to machines, products to factories, research problems to groups, vehicles and drivers to different routes etc. An issue of this nature is named an assignment problem.

II. Definition

Assignment problem may be a special kind of problem which deals with allocation of assorted resources to varied activities on one to 1 basis. It's tired such some way that the whole cost or time involved within the process is minimum or the whole profit is maximum.

III. Conditions

i) Number of jobs is capable number of machines or workers.

ii) Each worker or machine is assigned to just one job.

iii) Each worker or machine is independently capable of handling any job.

iv) Objective of the assignment is clearly specified (minimizing cost or maximizing profit)

3.1 Assignment Model:

Given n workers and n jobs with the price of each worker for each job, the matter is to assign each worker to at least one and only 1 job so on to optimize the full cost.

Let C_{ij} be the cost of assigning i^{th} to j^{th} job, x_{ij} be the assignment of i^{th} worker to j^{th} job and x_{ij} = 1, if i^{th} worker is assigned to j^{th} job = 0, otherwise

Following table represents the value of assigning n workers to n jobs.

Worker	Jobs					
	1	2	3	…	…	n
1	C_{11}	C_{12}	C_{13}			C_{1n}
2	C_{21}	C_{22}	C_{23}			C_{2n}
. . .	. . .	. . .	. . .	. . .	. . .	. . .
N	C_{n1}	C_{n2}	C_{n3}	…	…	C_{nn}

The objective is to form assignments that minimize the entire cost.

Thus, an assignment problem will be represented by n x n matrix which covers all the n! possible ways of constructing assignments.

Assignment Problem may be a special case of Applied Math Problem.

Assignment problem will be expressed symbolically as follows:

Minimize Z = $\sum_{i=1}^{n} \sum_{i=1}^{n} C_{ij} x_{ij}$

Subject to constraints

$\sum_{j=1}^{n} x_{ij} = 1; i = 1, 2, 3, \ldots n$

(exactly one job is assigned to i^{th} worker)

$\sum_{i=1}^{n} x_{ij} = 1; i = 1, 2, 3, \ldots n$

(exactly one worker is assigned to j^{th} job where x_{ij} takes a worth 0 or 1.

3.2 Hungarian method is predicated on the subsequent properties:

1) If a relentless (positive or negative) is added to each element of any row or column within the given cost matrix, an assignment that minimizes the overall cost within the original matrix also minimizes the overall cost within the revised matrix.

2) In an assignment problem, an answer having zero total cost of assignment is an optimal solution.

The Hungarian algorithm is explained with the assistance of the subsequent example.

Consider an example where 4 jobs have to be performed by 4 workers, one job per worker. The matrix below shows the value of assigning a specific worker to a particular job. The target is to attenuate the whole cost of assignment.

Workers	Jobs			
	J_1	J_2	J_3	J_4
W_1	62	63	50	72
W_2	57	35	49	60
W_3	21	49	15	56
W_4	18	19	78	23

3.2 Let us solve this problem by Hungarian method.

Step 1: Subtract the littlest element of every row from every element of that row.

Workers	Jobs			
	J_1	J_2	J_3	J_4
W_1	12	13	0	22
W_2	22	0	14	25
W_3	6	34	0	41
W_4	0	1	60	5

Step 2: Subtract the littlest element of every column from every element of that column.

Workers	Jobs			
	J_1	J_2	J_3	J_4
W_1	12	13	0	17
W_2	22	0	14	20
W_3	6	34	0	36
W_4	0	1	60	0

Step 3: Assign through zeros.

Workers	Jobs				
			√		
W_1	12	13	(0)	17	√
W_2	22	(0)	14	20	
W_3	6	34	~~0~~	36	√
W_4	(0)	1	60	~~0~~	

Observe that third row doesn't contain an assignment.

Step 4: 1. Mark (√) the row (R_3).

2. Mark (√) the columns (C_3) having zeros within the marked rows.

3. Mark (√) the row (R_3) which contains assignment in marked column.

4. Draw lines through the marked columns and unmarked rows.

Workers	Jobs				
			√		√
W_1	12	13	(0)	17	
W_2	~~22~~	~~(0)~~	~~14~~	~~20~~	√
W_3	6	34	~~0~~	36	
W_4	~~(0)~~	~~1~~	~~60~~	~~0~~	

All zeros will be covered using 3 lines

So, number of lines = 3 and order of matrix = 4

Hence, the quantity of lines required ≠ order of matrix.

Therefore, we continue with the subsequent step to form additional zeros.

Step 4: (i) Find the littlest uncovered element (6)

(ii) Subtract this number

3.3 Steps of the Hungarian Method:

Following steps describe the Hungarian Method.

Step 1. Subtract the minimum cost in each row of the price matrix from all the weather within the respective row.

Step 2. Subtract the minimum cost in each column of the value matrix from all the weather in the respective column

Step 3. Starting with the primary row, check the rows step by step until a row containing exactly single zero is found. Make an assignment by marking (□) that zero. Then cross (×) all other zeros within the column within which the assignment was made. This eliminates the livelihood of constructing further assignments in this column.

Step 4. After examining all the rows, repeat the same procedure for columns. i.e. examine the columns one by one until a column containing exactly one zero is found. Make an assignment by marking (□) that zero. Then cross (×) all other zeros within the row within which the assignment was made.

Step 5. Continue these successive operations on rows and columns until all the zeros are either assigned or crossed out and there's exactly one assignment for each row and each column. In such case optimal solution is obtained.

Step 6. There could be some rows (or columns) without assignments i.e. the entire number of marked zeros is a smaller amount than the order of the price matrix. In such case, proceed to step 7.

Step 7. Draw the smallest amount possible number of horizontal and vertical lines to hide all zeros.

This can be done as follows:

i) Mark ( ) the rows within which no assignment has been made.

ii) Mark ( ) the column having zeros within the marked rows.

iii) Mark ( ) rows which contain assignments in marked columns.

iv) Repeat 2 and three until the chain of marking is completed.

v) Draw straight lines through marked columns.

vi) Draw straight lines through unmarked rows.

By this fashion we draw the minimum number of horizontal and vertical lines required to cover all zeros. If the amount of lines is less than the order of matrix, then there's no solution. And if the minimum number of lines is adequate to the order of matrix, then there is an answer and it's optimal.

Step 8. If minimum number of lines < order of matrix, then

a) Select the littlest element not covered by any of the lines of the table.

3.4 Solve the following assignment using Hungarian Assignment Method:

Operator	Machine				
	I	II	III	IV	V
1	18	24	19	20	23
2	19	21	20	18	22
3	22	23	20	21	23
4	20	18	21	19	19
5	18	22	23	22	21

Step 1: Subtract the smallest element of each row from each element of that row.

	I	II	III	IV	V
1	0	6	1	2	4
2	1	3	2	0	3
3	2	3	0	1	3
4	2	0	3	1	1
5	0	4	5	4	3

Step 2: Subtract the smallest element of column from each element of that column.

	I	II	III	IV	V
1	0	6	1	2	4
2	1	3	2	0	3
3	2	3	0	1	2
4	2	0	3	1	0
5	0	4	5	4	2

Step 3: Draw minimum number of lines (horizontal and vertical) that are required to cover all zeros in the matrix.

	I	II	III	IV	V	
1	0	6	1	2	4	√
2	1	3	2	0	3	
3	2	3	0	1	2	
4	2	0	3	1	0	
5	0	4	5	4	2	√

Here, minimum number of lines (4) < order of matrix (5). Therefore, we continue with the next step to create additional zeros.

Step 4: (i) Find the smallest uncovered element (1)

(ii) Subtract this number from all uncovered elements and add it to all elements which lie at the intersection of two lines.

	I	II	III	IV	V	
1	0	5	0	1	3	√
2	2	3	2	0	3	
3	3	2	0	1	2	√
4	3	0	3	1	0	
5	0	3	4	3	1	√

Here, minimum number of lines (4) < order of matrix (5). Therefore, we continue with the next step to create additional zeros.

Step 5: (i) Find the smallest uncovered element (1)

(ii) Subtract this number from all uncovered elements and add it to all elements which lie at the intersection of two lines.

(iii) Then assign through zeros.

	I	II	III	IV	V
1	0	4	0	0	2
2	4	3	3	0	3
3	3	1	0	0	1
4	4	0	4	0	0
5	0	2	4	2	0

Optimal Solution: 1→I, 2→IV, 3→III, 4→II, 5→V

Minimum Value = 18 + 18 + 20 +18 + 21 = 95

IV. Conclusion

In this paper, we explained the proposed algorithm and, by numerical illustration, demonstrated its effectiveness. And we get the optimal solution, which is the same as the HA-method and MOA-method optimal solutions. This paper therefore presents a new approach that is easy to address the issue of assignment.

V. References

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