

VIVA Institute of Technology

Department of Applied Mathematics

All divisions(Sem-II)

Academic year: 2014-15

Assignment no 5

1

1. Show that

$$\int_0^{\infty} \sqrt[4]{x} e^{-\sqrt{x}} dx = \frac{3\sqrt{\pi}}{2}$$

2

P.T
$$\int_0^{\infty} x e^{-x^8} dx \cdot \int_0^{\infty} x^2 e^{-x^4} dx = \frac{\pi}{16\sqrt{2}}$$

3

Show that
$$\int_0^1 \frac{dx}{\sqrt{x \log \frac{1}{x}}} = \sqrt{2\pi}$$

4

Show that
$$\int_0^1 x^{a-1} \left(\log \frac{1}{x} \right)^{n-1} dx$$

5

Prove that $B(m, n) = B(m, n+1) + B(m+1, n)$

6

Show that (i)
$$\int_0^1 \sqrt{(\sqrt{x} - x)} dx = \frac{\pi}{8}$$

7

Show that
$$\int_0^{\pi/2} \sin^m \theta d\theta \cdot \int_0^{\pi/2} \sin^{m+1} \theta d\theta = \frac{\pi}{2(m+1)}$$

8

Evaluate
$$\int_0^1 \frac{x^2 + x^3}{(1+x)^7} dx$$

9

Prove that
$$\int_0^1 \frac{x^{2n}}{\sqrt{1-x^2}} dx = \frac{(2n)!}{2^{2n} (n!)^2} \frac{\pi}{2}, \text{ where } n \text{ is a positive integer}$$

10

Prove that
$$B(m, m) \cdot B\left(m + \frac{1}{2}, m + \frac{1}{2}\right) = \frac{\pi}{m} 2^{1-4m}$$